

# **An Informal Introduction to**

## **Formal Methods in Robotics**



Presented by Abhishek Kulkarni

# What this is NOT about!

- Comprehensive Coverage of Topics in Formal Methods in Robotics
- Explanation of Prerequisites





# Organization of Talk

## Sections

- What can Formal Methods do?
- The Formal Methods...
- How Formal Methods work?
- Where to use Formal Methods?
- Open Problems
- Meet few other friends... (if time permits!)



# Section I

## What can Formal Methods Do?



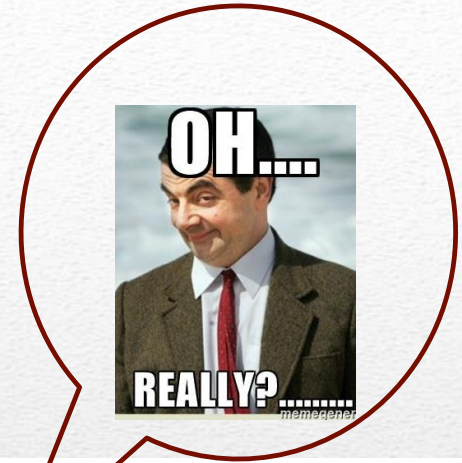
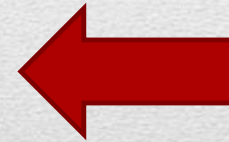
# Have you wondered?

- Why your laptops work so reliably?
- Why your bank never misplaces a dime?
- If you can talk with your robot in English Language



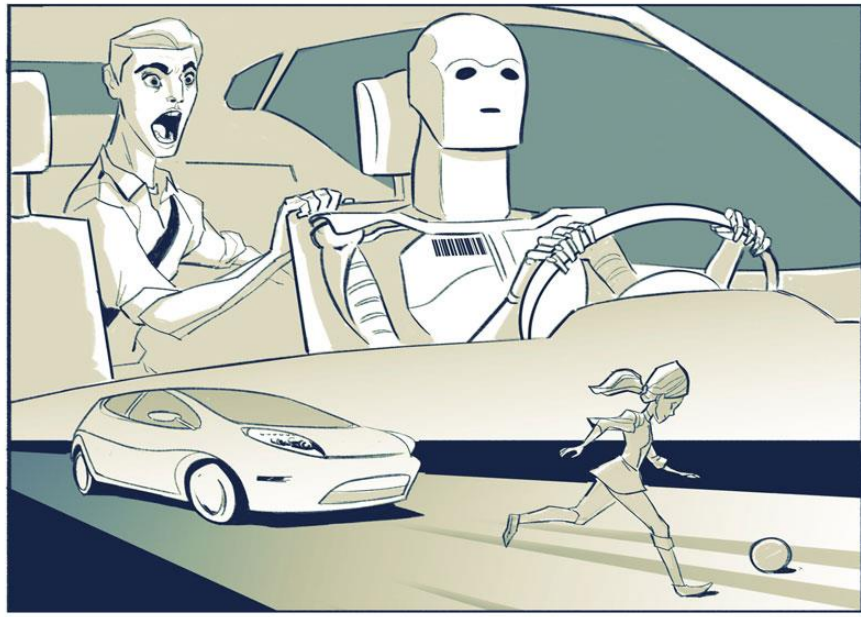


# The Big Question...





# How to Avoid this?



**COSTLY MISTAKE!!**



**SUPER-INTELLIGENCE?**

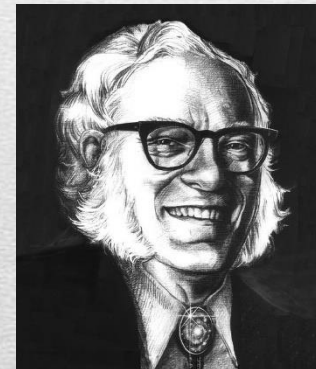
Reference: Robohub ([Link](#)) and I, Robot ([Link](#))



# Formal Methods



- Guarantee Satisfaction of Specifications
  - Automatic Policy Synthesis
  - Talk to Robot in (Almost) English!
- 
- Asimov is Happy!
    - Don't Kill Humans is Guaranteed 100%





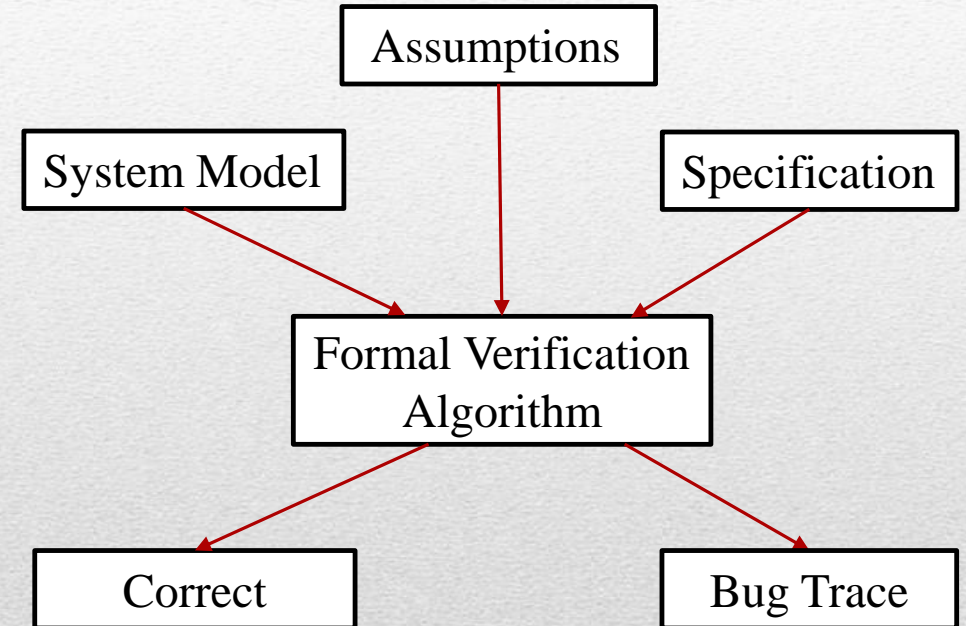
# Section II

## The Formal Methods...



# Verification Problem

- We know
  - How world looks – Model
  - What robot should do – Specifications
- Question: Does any action-sequence of robot satisfy the specification?
- Remember Remember... Laptops!





# Can I Always Win?

- Classic Tic-Tac-Toe

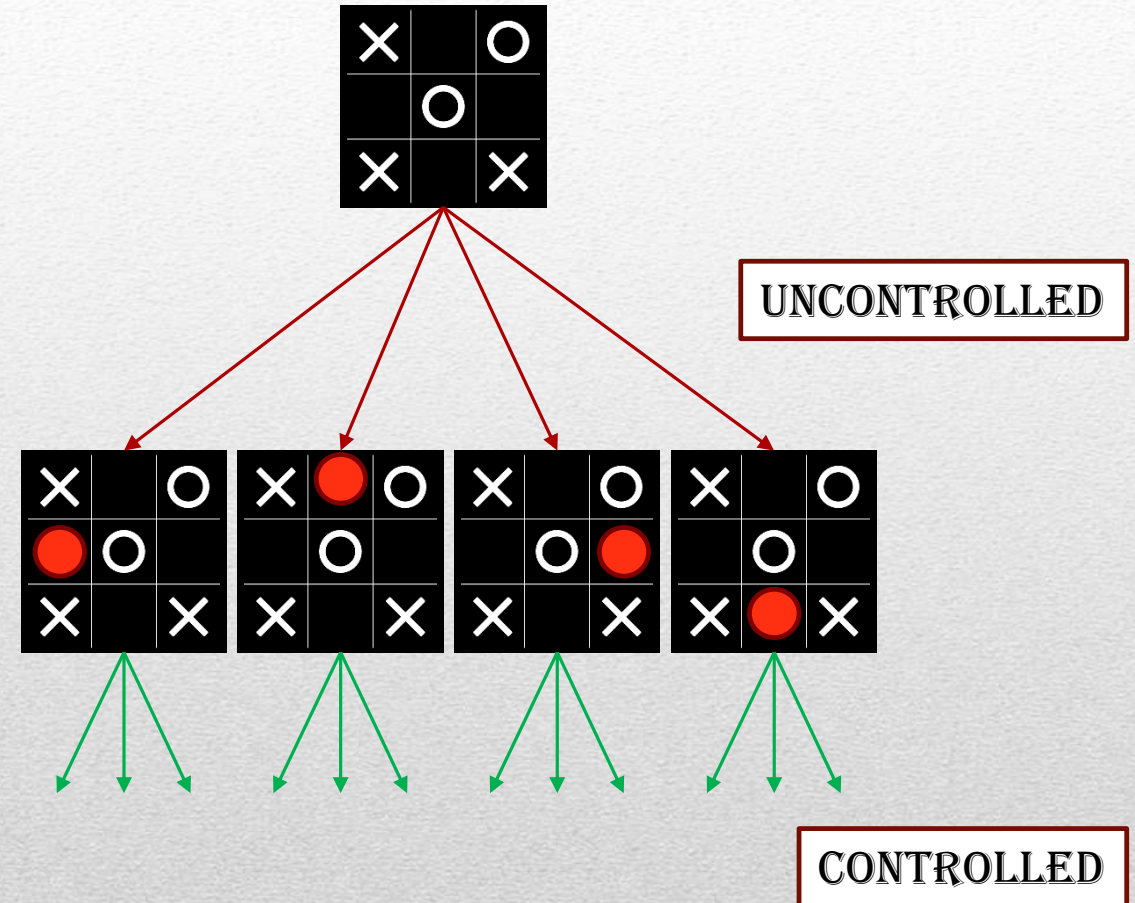
**No matter what my opponent does,  
Can I win?**

- Specification:

**Reach at least 1 of winning states!**

- Verification tells us: **What's guaranteed!**
- AND returns:

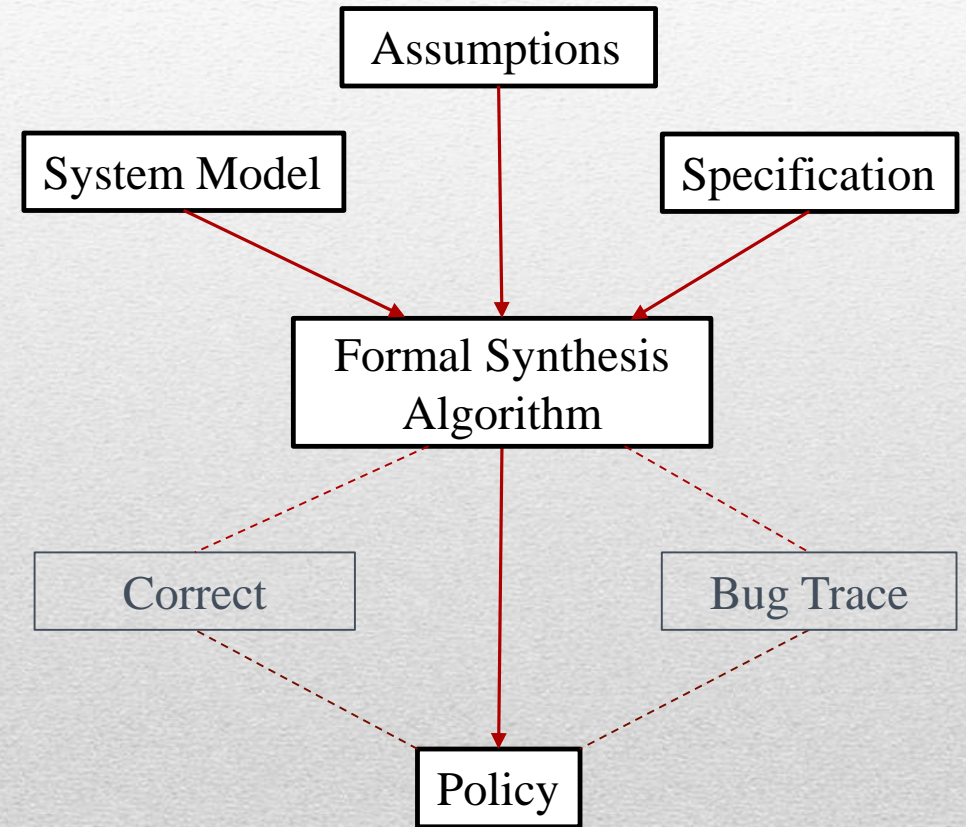
**In what ways I can loose...**





# Synthesis Problem

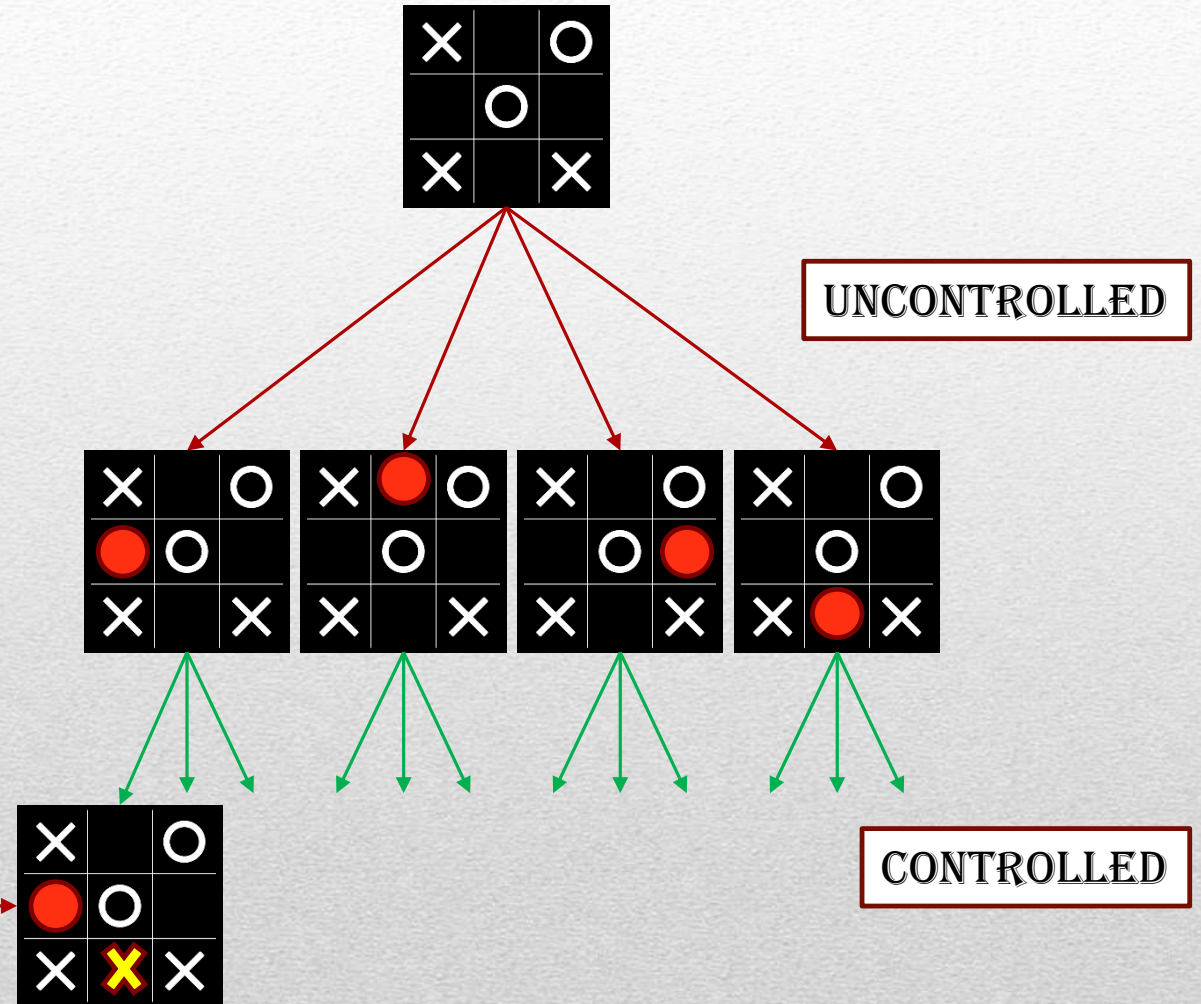
- We know
  - How world looks – Model
  - What robot should do – Specifications
- Question: What should robot do?





# Tell me... How to Win!

- Classic Tic-Tac-Toe  
No matter what my opponent does,  
Can I win?
- Specification:  
Reach at least 1 of winning states!
- Synthesis tells us:





# Section III

## How Formal Methods Work?



# Communication

- Language is fundamental to communication!
- Don't know some language – Interpreter
- Compilers!



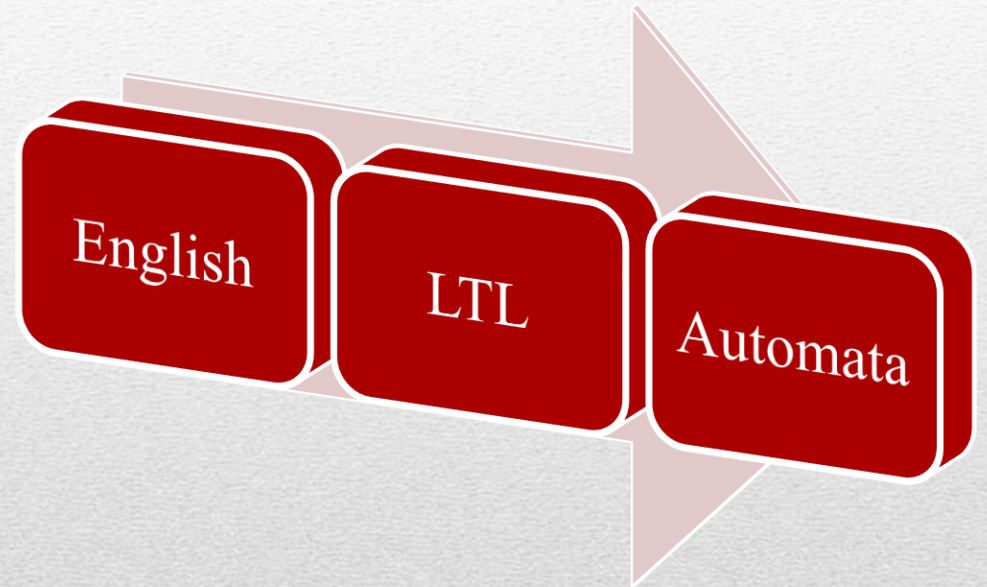
Image Reference: clip-art library ([Link](#))



# Linear Temporal Logic (LTL)

LTL makes Humans and Robot's Happy!

- Human Language = Too many to write!
- Robot's Language = Automata!
- LTL is almost as expressive as English!
- Beauty: LTL  $\rightarrow$  Automata is easy!





# Linear Temporal Logic (LTL)

LTL makes Humans and Robot's Happy!

Example 1:

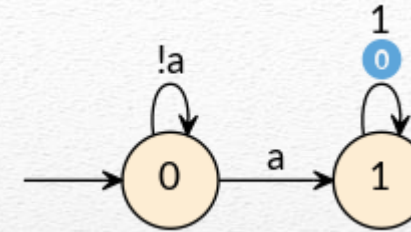
- English: Go to Living Room
- $\sim$ LTL: Eventually (Robot in Living Room)
- LTL:  $\diamond (state == living\ room)$

Example 2:

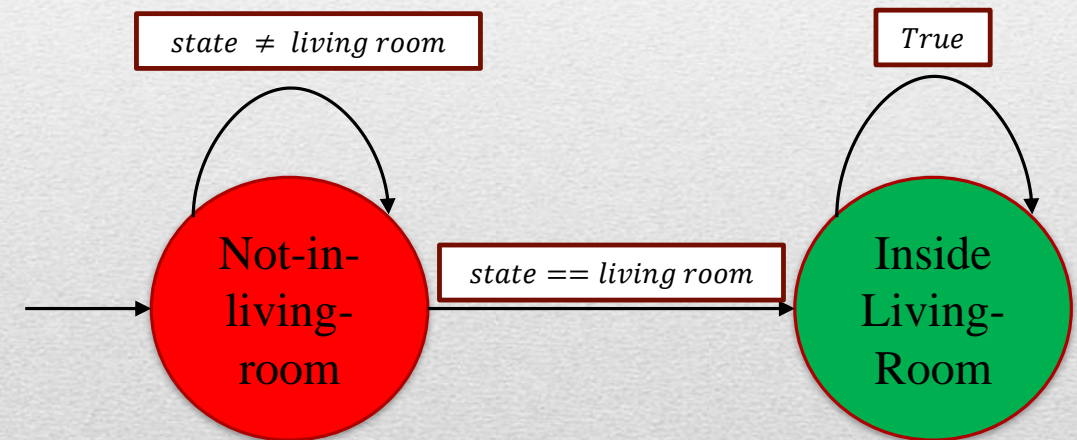
- English: Go to Living Room AND Never Collide with Obstacle
- $\sim$ LTL: Eventually (Robot in Living Room) AND Always (no-collision)
- LTL:  $\diamond (state == living\ room) \wedge \square ! is\_colliding(env, robot)$



# Final Step: Automaton



- Recall: Automata is robot's language!
- Robot wants to reach to a ***final state!***
- Communication is complete!



◇ ( $state == living\ room$ )



# How Robot Makes a Decision?

- **Model:** Understanding of world
  - Game Graph
  - Markov Decision Process
- **Objective:** Reach the Goal!
- **Easy Part:** Robot can choose what it want's to do!
- **Difficult Part:** It can't control what environment might do!

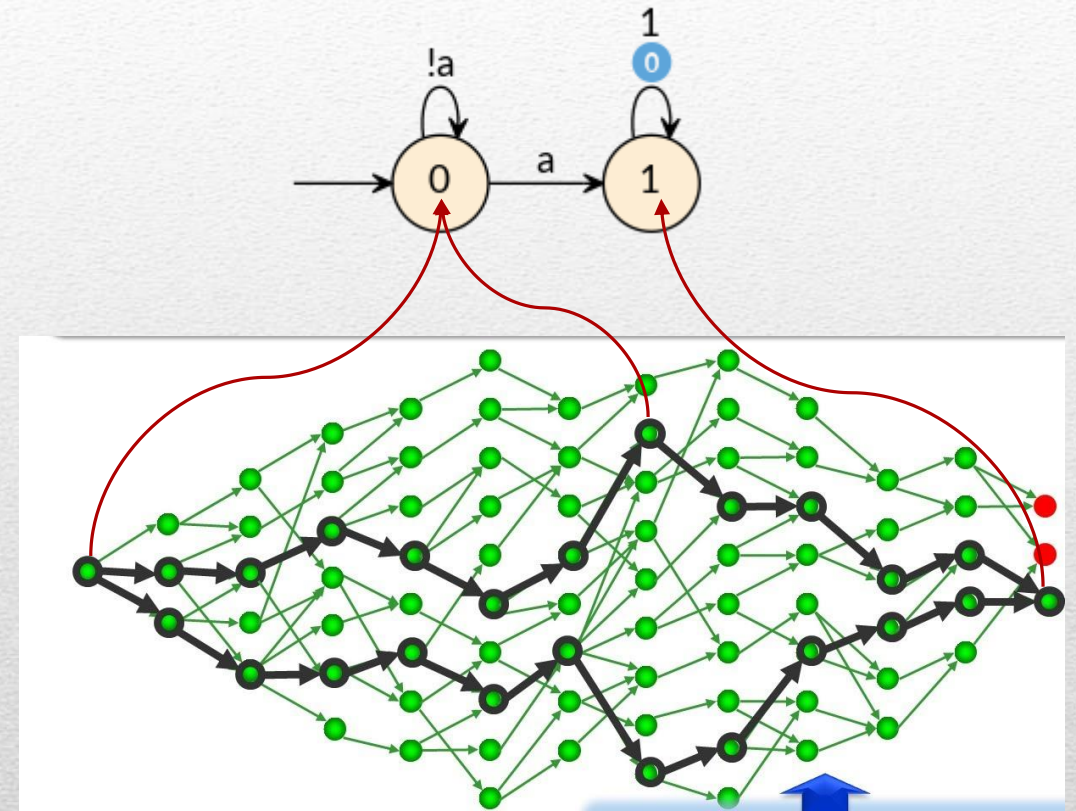
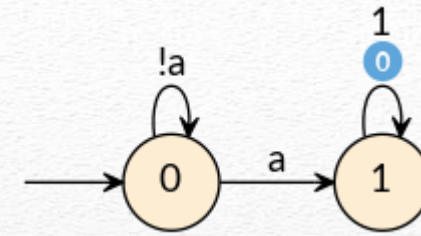


Image Reference: Mentor Graphics Questa Platform Video ([Link](#))

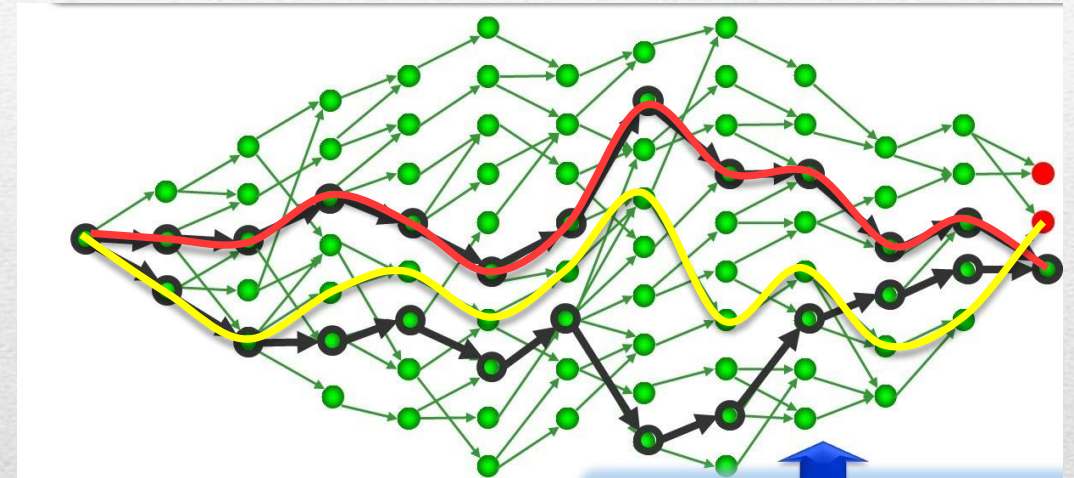


# The beauty of FM

- It's a BAD IDEA to find and check all paths from start to end!
- GOOD IDEA: Somehow compress a complete path into a state!
- That's a Product Operation.
- And that's how life becomes easy!



X



11

Composed  
State

Accept

Composed  
State

Reject

Image Reference: Mentor Graphics Questa Platform Video ([Link](#))



# Section IV

## Where to use Formal Methods?



# Application 1: Autopilots

- TAM Flight 402
  - Autopilot Engaged Reverse Thrusters Incorrectly
  - Airplane has 100's of sensors
  - Autopilot is modularly designed.
  - How do we detect – accidental dependencies?
- Idea: Specify using (English-like) LTL what **shouldn't** happen with aircraft at any time



Video Reference: Air Crash Investigation Series



# Application 2: Surgical Robots

- **Objective 1:** Safety – MUST ALWAYS HOLD!
  - Don't let needle move outside a particular volume
  - Or patient is history!
- **Objective 2:** Liveness – Satisfy if possible
  - Insert needle within a given area
  - And inject the medicine
- FMR allows full controller synthesis!

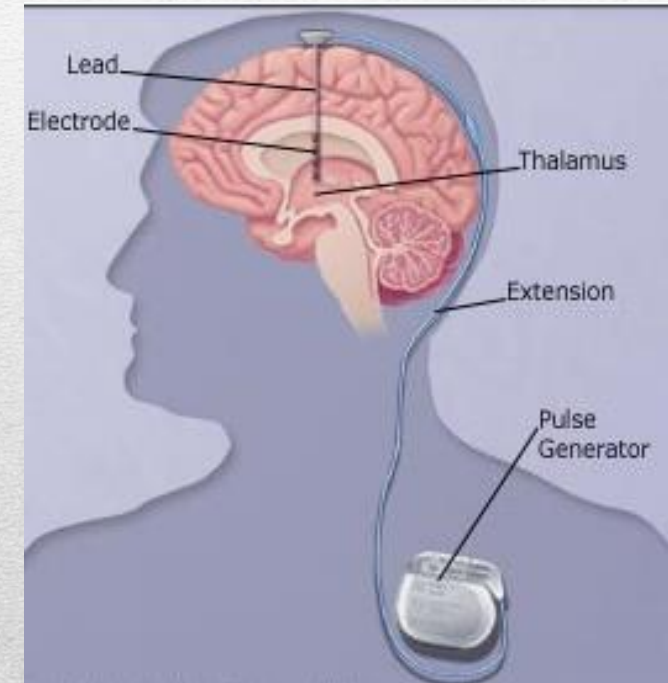


Image Reference: WebMD, Inc. ([Link](#))



# Section V

## Open Problems



# Theoretical

- Thanks to Mathematicians and Computer Scientists in 50-100 years!
- **Problem 1:** State Explosion
  - Recall Product Operation ([Link](#))
- **Problem 2:** Parallelism
  - Mathematicians had different aims for FM development.
  - Roboticians want speed and accuracy!
  - We need to develop parallel algorithms that are provably-correct



# Extending FMR

- Majority FMR studies are with
  - Linear Temporal Logic **or** Dynamic Differential Logic
  - Zero-Sum Games
- Problem 1: Different Logics
  - Explore use of different logics like Probabilistic Logics.
  - If you love physics – Try Quantum Logic!
- Problem 2:
  - Generally, cars on highway are not your enemies. But they have objective and can be aggressive! This is different game
  - Use FMR with a non-zero sum game models.



# Apply FMR

- Majority FMR researchers are theoretical! Thrust something into reality!
- **Problem 1:** Toolkits
  - Develop Generalized and Efficient Toolkits
  - Modular, Abstract!
- **Problem 2:** Cyber-Physical Systems
  - Control – Automatic Controller Synthesis for Autonomous Car, Quadrotors etc.
  - Coordination – Multi-Agent Behaviors, Platoons etc.



# Section V

## Meet some friends...



# Concept 1: Abstraction

- Robot may reason at higher level – like Behaviors
- How can robot understand what behavior to use?
- Abstraction is mathematical tool to do this!

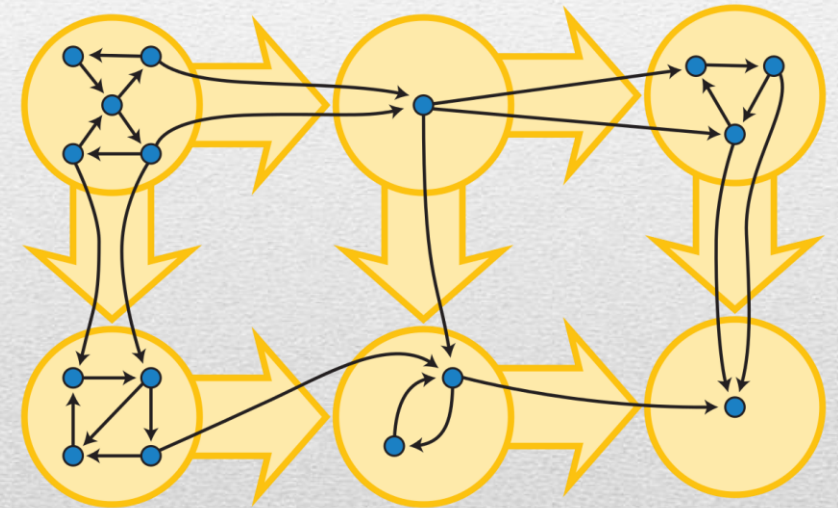
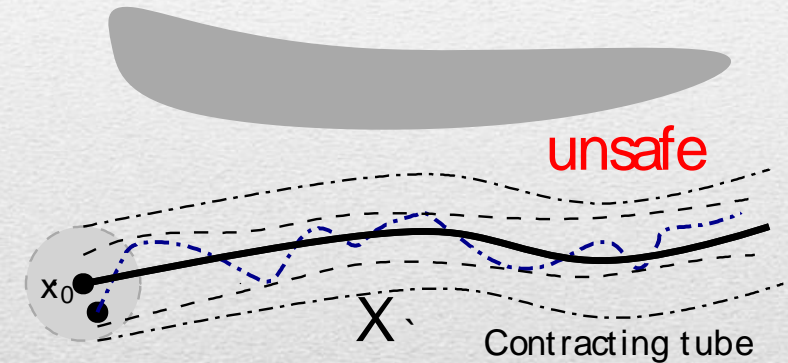


Image Reference: Wikipedia ([Link](#))



# Concept 2: Contraction Theory

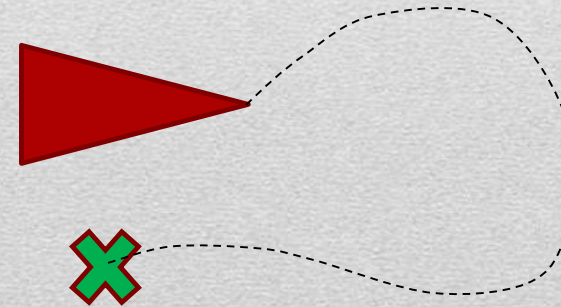
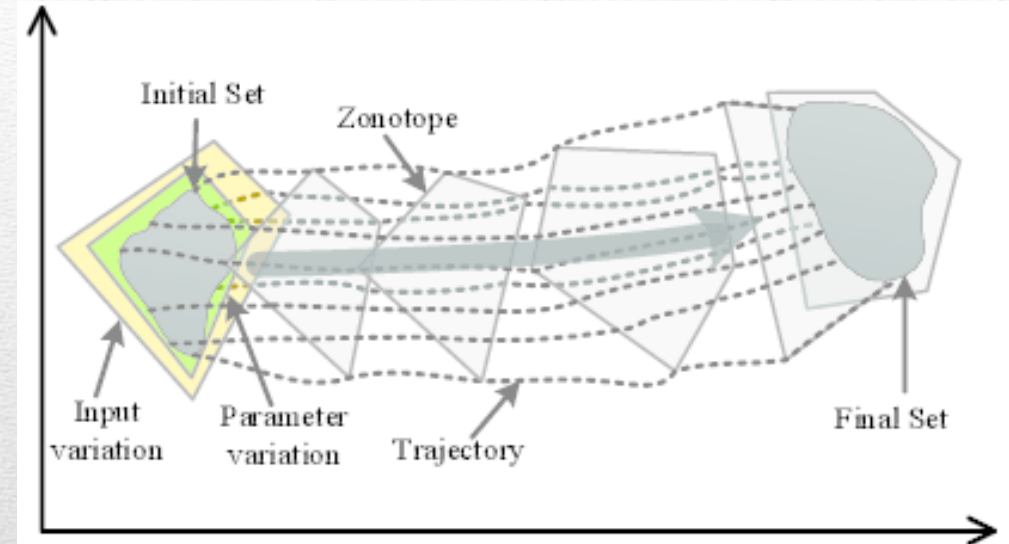
- Allows checking stability of trajectories!
- Intuition: Binary Search
  - Sandwich from above and below
  - And say Hurray!





# Concept 3: Reachability

- Intuition:
  - As time progresses...
  - Which all points can I reach?
- Example:
  - Green point is not reachable in 1-step for the robot





# Disclaimer



Ask Questions!

Thank You!